Anatomy of a Performance Task Sample: Merry-Go-Round

1) Name and source of the item or task
Merry-Go-Round, Grade 7, Shelbi Cole, CT State Department of Education
2) The item/task sample

Non-Scorable Summative Component (Day 1)

Materials:

- Masking tape or Painter's tape
- Protractor
- Tape measure or ruler

Interactions and Planning (prior to administration of scored component): A certified classroom teacher provides a lead-in to the performance task.

Inform students that they will be completing a mathematics task over the next couple of days that asks them to provide blueprints for a playground merry-go-round in the shape of a regular octagon. Prior to this task, the class must complete some important planning steps.

1) Teacher and students work together to develop a common understanding of the term "merry-go-round" and specifically, the "platform" of the merry-go-round as it is used in a playground blueprint. The links below provide useful photographs and information to help guide a discussion of these two terms that are essential prior knowledge for the upcoming performance task. It is important that students understand merry-go-round in the context of a playground, rather than as what is traditionally called a "carousel."
   http://www.indiamart.com/honeyfunnthrill/playground-merry-go-round.html
   http://www.unc.edu/~sonyah/merrygorounds.htm

2) Complete the learning task.
   Create ample space in the front of the classroom by clearing furniture to the sides of the room, or find an alternate location. Work with students to create an acceptable range for the side lengths of a regular octagonal real-life merry-go-round using the following steps:

- Ask for two students to volunteer to be on the smallest possible merry-go-round. These students can sit in chairs or on the floor as the outline for a regular octagon is built around them using masking or Painter's tape. Work with students to use the tape to create an eight-sided outline with 135° interior angles between adjacent sides. [This does not have to be a perfect representation. Students need an idea about a good minimum side length.] Record the side length, in inches, of the merry-go-round, which will serve as the minimum side length for students to use in the subsequent performance task.
- Repeat the task of constructing a regular octagon with tape, this time with eight student volunteers sitting close together as though they are on a large merry-go-round. Once the outline has been created, record the side length of the octagon, in inches, for the upper limit for possible side lengths of a regular octagonal merry-go-round.

3) Use the following questions to engage the class in the context and reinforce important concepts for the upcoming task.

- What factors were important in making our decision that the merry-go-round could not have sides any longer and/or shorter? Give a specific example of what might happen if your design was any bigger and/or smaller.
- How many people can be seated safely on the merry-go-round outlined in each case?
- What is the measure of each interior angle in the large regular octagonal merry-go-round? What is the measure of each interior angle in the smaller regular octagonal merry-go-round? Ask students to draw an octagon with interior angle measures that are not all equal to 135°.
- Use the information obtained from the activity to develop a "class range" for the acceptable measures of the side lengths of a regular octagonal merry-go-round and write the range as a compound inequality. [For example, 3 ft $\leq s \leq$ 8 ft, where *s* is the side length of a playground's regular octagonal merry-go-round.]

Scorable Summative Component

Materials:

- Ruler
- Protractor
- Calculator
- Mathematical drawing compass
- 3 Sheets of plain white drawing paper, standard size

Teacher MUST display this information for students from the non-scorable component for use in this component:

1) The range of acceptable values for the side lengths of a regular octagonal merry-go-round
2) Measure of each angle of a regular octagon = 135°

The Situation: Plans for a new playground include a regular octagonal merry-go-round. The planning committee has requested a report that includes scale drawings of two different design sizes for the octagonal platform that adhere to the design and safety requirements outlined in the *Public Playground Safety Manual.* The committee has also requested calculations for the area of each potential design to appropriately place the equipment into its playground blueprint.

Your Task (Time Block 1 – 120 minutes): Create scale drawings of two different regular octagonal design merry-go-rounds and a comparable circular design to deliver to the committee. Include calculations for the areas of each. Use the following steps to guide this process:

Comment [SC1]: The computer will deliver the instructions to the student in a series of steps, delineated by separate bullet points.

- Select two different side lengths that can be used to create two different regular octagonal merry-go-round platforms, based on the range of acceptable side lengths selected by your class. Choose the two side lengths such that one is at least one inch longer than the other. [ENTER side lengths into computer]

- Beginning with the longer of the two side lengths, determine a scale factor that will allow you to draw a model of the entire regular octagonal merry-go-round platform on a standard 8 ½ x 11" sheet of paper. Apply the scale factor to create a scale drawing, using precise measurements for both side lengths and angles. [ENTER scale factor and scan/upload drawing.] If the scale factor you have selected *does not* allow you to create the entire drawing on the paper, DO NOT START OVER. Explain in the box provided why the scale factor did not work and give a scale factor that would have been a better choice. If the scale factor DID allow you to create your entire drawing, explain how you selected the scale factor to ensure it would fit.

  Comment [c2]: Possibility exists to build some of this into technology, such as through the use of a geometry tool application.

  - Calculate the area of the regular octagonal platform in square inches using the real-life side length. [Set computer to display the side length student entered on prior screen so he/she does not use the scale measurement. Student will enter the area calculated into BOX 1 and explain or show work in BOX 2].

- Applying the same scale factor, create a drawing of the second regular octagonal platform on a separate 8 ½ x 11" sheet of paper, using precise measurements for both side lengths and angles. [ENTER scale factor and scan/upload drawing.]

  - Calculate the area of the regular octagonal platform in square inches using the real-life side length. [Set computer to display the side length student entered on prior screen so he/she does not use the scale measurement. Student will enter the area calculated into BOX 1 and explain or show work in BOX 2].

- According to the Handbook for Public Playground Safety, "The difference between the minimum and maximum radii of a non-circular platform should not exceed 2.0 inches" (Handbook for Public Playground Safety, p. 31). [See Figure 15 on page 31 for definitions of minimum and maximum radii.]

  For each regular octagonal platform, determine whether the difference between the minimum radius and maximum radius meets this requirement. Be sure to calculate the difference for the real dimensions, not the scale dimensions. Show all work or explain how you determined your answer. [Students *may* draw/write on their scale drawings if they are using them to complete this task, but may also choose methods that do not require these drawings. They will enter certain values related to this task into computer.]

- Using the maximum radius of your larger octagonal platform design and the same scale factor [display values student entered from prior work for these two features on computer screen], create a scale drawing of a circular platform design.
  - Calculate the area of the circular platform in square inches using the real-life radius. [Set computer to display the radius so he/she does not use the scale measurement. Student will enter the area calculated into BOX 1 and explain or show work in BOX 2].

- Which had a larger area, the octagonal platform with maximum radius ___, or the circular design with the same radius? Will this be the case anytime the radius of the circle equals the maximum radius of the octagon? Explain your reasoning.

3) Rubric or scoring criteria for the item/ task

The student's response has the following:

- Side lengths within specified range pre-determined by class during non-scorable component
- Scale factor that enables the entire octagon to be drawn on the paper with given dimensions (larger octagon only)
- Scale factor calculations that are accurate
- Scale drawings that are precise (angle measures, side lengths, and/or radii)
- Explanation for why scale factor was selected OR explanation for why the scale factor selected was not a good choice
- Precise calculations for areas of octagonal and circular platforms given side lengths and radii
- Clear mathematical evidence that the difference between the minimum and maximum radius is less than or equal to 2" for each octagonal platform
- A clear stand on whether a circle with radius equal to the maximum radius of a regular octagon will always, sometimes, or never have greater area and an explanation or examples that support this choice

4) Common Core Standards measured by this task (CCS standard number, description of standard)

Content Standards	Practice Standards
7.G.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. 7.G.2 Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. 7.G.4 Know the formulas for the area and circumference of a circle and use	SMP.1 Make sense of problems and persevere in solving them. SMP.2 Reason abstractly and quantitatively. SMP.3 Construct viable arguments and critique the reasoning of others. SMP.4 Model with mathematics. SMP.5 Use appropriate tools strategically. SMP.6 Attend to precision. SMP.7 Look for and make use of structure. SMP.8 Look for and express regularity in repeated reasoning.

them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. 7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	

5) Additional Comments about the task

List of Descriptive Features Required of All Performance Tasks –Use Comments to annotate the Performance Task above to show where each feature is included in the task.

Integrates knowledge and skills across multiple standards— This task incorporates four content standards and at least three standards for mathematical practice. Students determine and apply a scale factor to create scale drawings (7.G.1), the scale drawings have given conditions (they are regular – establishing conditions for angle measure and side length)(7.G.2), they use the formula for the area of a circle (7.G.3), and they apply one or more area formulas to find the octagonal areas (7.G.4). With respect to the Standards for Mathematical Practice, students have to make sense of the problem (SMP.1), use appropriate tools strategically (SMP.5) and attend to precision (SMP.6) to successfully complete this problem. Evidence of other standards is dependent on a selected method. For example, a student who divides the octagon into two trapezoids and a rectangle will show strong evidence of the use of structure (SMP.7).

Measures capacities such as depth of understanding, research skills and/or complex analysis with relevant evidence— Students are asked to use information from the Handbook for Public Playground Safety. Also, the final question requires an analysis that asks students to move from a specific case to a more general case, probably applying SMP.2, which asks them to make an abstract conclusion from more general, quantitative calculations.

Requires student-initiated planning, management of information and ideas, interaction with other materials and/or people—Students work in large groups to come to a consensus on an acceptable range of side lengths for a playground merry-go-round.

Reflects a real-world task and/or scenario-based problem—Students engage in the processes of blueprint creation and area planning, which are valuable skills in a wide variety of careers.

Lends itself to multiple approaches—Students select their own scale factors, can divide the octagon in multiple ways to apply known area formulas, and are able to use a variety of methods to find the difference between the minimum and maximum radius (which is more complex than it seems since the Pythagorean Theorem is not in the CCSS until Grade 8).

Represents content that is relevant & meaningful to students: The context combined with the integration of content and practice standards help make this task relevant and meaningful.

Allows for demonstration of important knowledge & skills, including those that address 21st century skills: The knowledge and skills assessed are important in several areas of mathematics and in several career paths.

Requires scoring that focuses on the essence of the task:

Reflects one or more of the Standards for Mathematical Practice, Reading and Writing, (or Speaking and Listening) Process: The expected SMPs are highlighted above.

Seems feasible for the school/classroom environment: Directions are provided to the teacher to make the task feasible for the classroom setting.